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Invariant surfaces in $\widetilde{PSL}_2(\mathbb{R},\tau)$ and applications. (English summary)

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In this paper, the author studies minimal surfaces and constant mean curvature surfaces, also called H-surfaces, that are immersed in the space $\widetilde{\mathrm{PSL}}_2(\mathbb{R},\tau)$ and that are invariant under a one-parameter group of isometries.

First, the definition of the universal covering of the projective special linear group $\widetilde{\mathrm{PSL}}_2(\mathbb{R},\tau)$ and the isometries and some properties of this space are recalled.

Then, rotational constant mean curvature surfaces, that is, constant mean curvature surfaces that are invariant under rotations, are characterized and some special examples are presented. The minimal rotational surfaces are studied in more detail: it is shown that when these surfaces are complete and embedded, they are called catenoids. The height of these catenoids is examined. When $H>0$, among the H-rotational surfaces in $\widetilde{\mathrm{PSL}}_2(\mathbb{R},\tau)$ analogues of the nodoids and the unduloids of Delaunay in $\mathbb{E}^3$ are encountered. Also, a half space theorem for $H=\frac{1}{2}$-rotational surfaces is proved.

Finally, some results on H-surfaces that are invariant under parabolic or under hyperbolic isometries in $\widetilde{\mathrm{PSL}}_2(\mathbb{R},\tau)$ and some of their geometric properties are mentioned.

Reference [13] to a preprint in this paper can now be completed: [R. Younes, Illinois J. Math. **54** (2010), no. 2, 671–712; MR2846478].

Reviewed by *Wendy Goemans*

References

1. U. Abresch and H. Rosenberg. *Generalized Hopf differential*. Math. Contemp., **28** (2005), 1–28 (electronic). MR2195187 (2006h:53004)
2. B. Daniel. *Isometric Immersions into 3-dimensional homogeneous manifolds*. Comment. Math. Helv., **82**(1) (2007), 87–131. MR2296059 (2008a:53058)
3. C. Figueroa, F. Mercuri and R. Pedrosa. *Invariant surfaces of the Heisenberg group*. Ann. Mat. Pura Appl., **177** (1999), 173–194. MR1747630 (2000m:53089)
4. L. Hauswirth, B. Daniel and P. Mira. *Constant mean curvature surfaces in homogeneous manifolds*. Kias Workshop on Differential Geometry (2009).
5. L. Mazet, M. Rodríguez and H. Rosenberg. *The Dirichlet problem for the minimal surface equation - with possible infinite boundary data - over domains in a Riemannian surface*, preprint.
6. B. Nelli, R. Sa Earp, W. Santos and E. Toubiana. *Uniqueness of H-surfaces in $\mathbb{H}^2\times\mathbb{R}|H|\leq 1/2$, with boundary one or two parallel horizontal circles*. Ann. Global Anal. Geom., **33**(4) (2008), 307–321. MR2395188 (2009k:53017)

7. R. Sa Earp. *Parabolic and hyperbolic screw motion surfaces in* $\mathbb{H}^2 \times \mathbb{R}$. J. Aust. Math. Soc., **85**(1) (2008), 113–143. MR2460869 (2010d:53067)
8. R. Sa Earp and B. Nelli. *A halfspace theorem for mean curvature* $H = 1/2$ *surfaces in* $\mathbb{H}^2 \times \mathbb{R}$. J. Math. Anal. Appl., **365**(1) (2010), 167–170. MR2585087 (2011b:53015)
9. R. Sa Earp and E. Toubiana. *An asymptotic theorem for minimal surfaces and existence results for minimal graphs in* $\mathbb{H}^2 \times \mathbb{R}$. Math. Ann., **342**(2) (2008), 309–331 (electronic). MR2425145 (2009k:53020)
10. R. Sa Earp and E. Toubiana. *Screw motion surfaces in* $\mathbb{H}^2 \times \mathbb{R}$ and $\mathbb{S}^2 \times \mathbb{R}$. Illinois J. Math., **49**(4) (2005), 1323–1362 (electronic). MR2210365 (2007m:53012)
11. E. Toubiana. *Note sur les variétés homogènes de dimension 3*.
12. W. Thurston. *Three-dimensional geometry and topology*. Princeton (1997).
13. R. Younes. *Minimal Surfaces in* $\widetilde{PSL}_2(\mathbb{R})$, prepint.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.